A Fuzzy Probability Algorithm for Evaluating the AP1000 Long Term Cooling System to Mitigate Large Break LOCA

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ABSTRACT
Components of nuclear power plants do not always have historical failure data to probabilistically evaluate their reliability characteristics. To overcome this drawback, an alternative approach has been proposed by involving experts to qualitatively justify basic event likelihood occurrences. However, expert judgments always involve epistemic uncertainty and this uncertainty needs to be quantified. Existing fault tree analysis quantifies uncertainty using Monte Carlo simulation, which is based on probability distributions. Since expert judgments are not described in probability distributions, Monte Carlo simulation is not appropriate for evaluating epistemic uncertainty. Therefore, a new approach needs to be developed to overcome this limitation. This study proposes a fuzzy probability algorithm to evaluate epistemic uncertainties in fault tree analysis. In the proposed algorithm, fuzzy probabilities are used to represent epistemic uncertainties of basic events, intermediate events, and the top event. To propagate and quantify epistemic uncertainty in fault tree analysis, a fuzzy multiplication rule and a fuzzy complementation rule are applied to substitute the AND Boolean and OR Boolean gates, respectively. To see the feasibility and applicability of the proposed algorithm, a case-based experiment on uncertainty evaluation of the AP1000 long term cooling system is discussed. The result shows that the best estimate probability to describe the failure of AP1000 long term cooling system generated by the proposed algorithm is 3.15×10^{-3}, which is very close to the reference value of 1.11×10^{-3}. This result confirms that the proposed algorithm offers a good alternative approach to quantify uncertainties in probabilistic safety assessment by fault tree analysis.

INTRODUCTION
The Westinghouse AP1000 is designed based on the previous generation of the Westinghouse AP600 by significantly increasing the power generation from 600 MWe to 1000 MWe. The United States Nuclear Regulatory Commission (US-NRC) and the European Utility Requirement have certified the AP1000 as a generation III+ nuclear power plant [1,2]. The AP1000 design is the first commercial nuclear power plant (NPP) design which implements passive safety systems [3]. Those passive safety systems function based on gravity, convection, condensation, and heat circulation [4-6]. Through the implementation of those passive safety systems, high reliability, human error minimization, simplification and easy modularization can be achieved [5,7]. However, passive safety systems may still fail due to the possibility of a false response to the physical phenomenon which it is based on. Therefore, the reliability of AP1000 passive safety system still needs to be evaluated.

The loss-of-coolant accident (LOCA) is defined as an accident in which coolant is freely discharge from the reactor primary system. Meanwhile, a large break LOCA is a design basis accident for pressurized water reactors
The AP1000 design provides three passive safety systems to mitigate the large break LOCA, i.e. injection system by accumulator (AI), low pressure injection system (LPI) which injects water from in-containment refuelling water storage tank (IRWST), and long term cooling system (LTCS) which injects water from passive containment cooling water storage tank (PCCWST) [9]. A number of valves, which will be automatically actuated to their safeguard positions when they lose power or receive an actuation signal, are aligned to those three passive safety systems.

Since the AP1000 is still under construction [6,10,11], historical failure data is still unavailable to statistically estimate basic event probabilities. Hence, the reliability of the AP1000 passive safety systems has to be studied by a fault tree analysis (FTA) using generic data [9,12-14]. Unfortunately, due to the use of those generic data, analysis results will not confirm the real performance of the AP1000 passive safety systems.

To overcome this limitation, Purba and Sony Tjahyani [15] have utilized experts to qualitatively justify basic event reliability characteristics of the AP1000 safety systems. Due to imperfect knowledge or incomplete information, epistemic uncertainties are always involved in experts judgements [16]. Zhou et al. [14] acknowledged that uncertainties in FTA need to be evaluated to achieve more reliable results.

Conventional FTA evaluates uncertainties using Monte Carlo Simulation. However, since Monte Carlo simulation is based on probability distribution, this technique is not appropriate for evaluating epistemic uncertainties raised in expert judgments, which do not come with probability distributions [17,18]. The motivation of this study is how to evaluate epistemic uncertainties in reliability study of the AP1000 passive safety systems due to the involvement of experts in the basic event reliability evaluations. Therefore, a fuzzy probability algorithm is developed by introducing fuzzy probabilities and fuzzy combination rules into fault tree analysis. Two aspects of originalities of this study are (1) an introduction of fuzzy probabilities into fault tree analysis to describe the occurrence likelihoods of basic events, intermediate events and the top event; and (2) a substitution of Boolean algebra by fuzzy combination rules to quantify fault trees. This study is the further development of our previous studies in [15,19].

THEORY

Expert elicitation

Expert elicitation can be defined as a structured process to formalize and quantify an uncertain quantity due to the limitations in the data or when such data is unattainable because of physical constraints or lack of resources [20,21]. This elicitation technique consists of three parts, i.e. experts whose expertises are related to the field being studied, justification on the event occurrence likelihoods, and analysts who will use the expert judgements. The expert elicitation process may integrate empirical data with scientific judgment and identify a range of possible likelihoods.

To properly characterize various factors that contribute to the overall uncertainty in the expert judgments, expert elicitation should be done in a panel but every individual expert in the panel should be elicited separately using a standardized protocol. Hence, the most robust picture of uncertainties can be achieved. To minimize a very wide spectrum of responses, experts to be elicited should be properly selected. Cooke et al. [22] recommended three indicators to select the most capable experts, i.e. number of scientific publications, recommendations from a wide range of experts, and experiences with previous similar studies. By scoring each criteria and summing up the total score, the experts whose expertise are more relevant to the study what it is intended for will be properly selected.

Unfortunately, in real-world applications, the experts may have different levels of expertise, backgrounds, and working experiences. Moreover, different scientific intuition and ability to integrate information with theories can also play critical roles in expert judgments. Therefore, experts could have different judgments on a same event. To accommodate these different judgments, it is important to aggregate those judgments into a single value to find a consensus.

Combining expert judgments requires relative weights of individual experts to each other. Different justification weights from 0 to 1 may be assigned to every expert to represent their credentials, credibility, or competency. An expert with a weight
of 1 is the most credible, while an expert with a lower weight is deemed to be less credible.

Cooke and Goossens [23] have formulated two key performance-based indicators, i.e. calibration and informativeness, to weight selected experts. For calibration process, ‘seed variables’ need to be provided in advance. Seed variables are variables whose values have been already known to the safety analysts but at the time of assessment the experts do not know those values. Using several calibration questions, the probabilities of experts to correctly answer the questions can be drawn and weight can be properly given to each expert. It is also important to note that the seed variables and the calibration questions must be as close as possible to the problems that the study is intended to solve [24].

CALCULATION METHODS

Fuzzy probability algorithm

A fuzzy probability algorithm is proposed in this study to evaluate epistemic uncertainty of the AP1000 long term cooling system to mitigate large break LOCA. The algorithm introduces fuzzy probabilities and fuzzy combination rules into fault tree analysis. A fuzzy probability is a membership function of fuzzy numbers used to describe the occurrence likelihood of an event. In this proposed fuzzy probability algorithm, fuzzy probabilities are represented by triangular fuzzy numbers and used to describe the likelihood of the occurrences of basic events, intermediate events, and the top event. To quantify the system fault tree, two fuzzy combination rules, i.e. a fuzzy multiplication rule and a fuzzy complementation rule, are introduced. A fuzzy multiplication rule quantifies the output of an AND Boolean gate, while a fuzzy complementation rule quantifies the output of an OR Boolean gate. The probability of the top event is then generated from the calculated top event fuzzy probability using a logarithmic function. The proposed fuzzy probability algorithm consists of three main steps. Each step is explained in details below.

Step 1: Generating basic event fuzzy probabilities

The objective of this step is to generate best estimate, lower bound, and upper bound fuzzy probabilities of basic events. To realize this objective, we have developed a set of seven failure possibilities to qualitatively describe the occurrence likelihoods of basic events related to the operation of nuclear power plants [25]. This set and its corresponding fuzzy probabilities are shown in Table 1.

<table>
<thead>
<tr>
<th>Failure possibilities</th>
<th>Fuzzy probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low (VL)</td>
<td>μVL(x) = (0.00, 0.04, 0.08)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>μL(x) = (0.07, 0.13, 0.19)</td>
</tr>
<tr>
<td>Reasonably Low (RL)</td>
<td>μRL(x) = (0.17, 0.27, 0.37)</td>
</tr>
<tr>
<td>Moderate (M)</td>
<td>μM(x) = (0.35, 0.50, 0.65)</td>
</tr>
<tr>
<td>Reasonably High (RH)</td>
<td>μRH(x) = (0.63, 0.73, 0.83)</td>
</tr>
<tr>
<td>High (H)</td>
<td>μH(x) = (0.81, 0.87, 0.93)</td>
</tr>
<tr>
<td>Very High (VH)</td>
<td>μVH(x) = (0.92, 0.96, 1.00)</td>
</tr>
</tbody>
</table>

In this step, experts, who have been selected and properly weighted using the elicitation technique described in the previous section, individually characterize a failure possibility of a basic event by responding to, for example,:

What is the failure possibility of basic event e?
Is it very low, low, reasonably low, moderate, reasonably high, high, or very high?

Using Table 1, the n failure possibilities given by n experts to a basic event e are then converted into their n corresponding fuzzy probabilities. Finally, the best estimate, lower bound, and upper bound fuzzy probabilities of basic event e are calculated as follows:

Step 1.1: Best estimate fuzzy probability calculation

The best estimate fuzzy probability of basic event e, which is represented by superscript \(\hat{b}\), is calculated by aggregating those n fuzzy probabilities into one fuzzy probability using (1).

\[ \mu_e^{\hat{b}}(x) = \sum_{i=1}^{7} w_j \times \mu_{ije}(x) \]  

(1)

where \(w_j\) is the weight of the \(j^{th}\) expert, \(\mu_{ije}(x)\) is the \(i^{th}\) fuzzy probability in Table 1 given by the \(j^{th}\) expert to basic event e, \(7\) is the number of fuzzy probabilities in Table 1, and \(n\) is the number of experts in the panel. \(w_j\) and \(\mu_{ije}(x)\) in (1) must meet properties defined in (2) and (3), respectively.

\[ 0 \leq w_j \leq 1; \sum_{j=1}^{n} w_j = 1 \]  

(2)

\[ \mu_{ije}(x) \in \{\mu_{VL}(x),\mu_{L}(x),\mu_{RL}(x),\mu_{M}(x),\mu_{RH}(x),\mu_{H}(x),\mu_{VH}(x)\} \]  

(3)
Step 1.2: Lower bound fuzzy probability calculation

The lower bound fuzzy probability of basic event \( e \) is generated using the lowest fuzzy probability from the \( n \) fuzzy probabilities assigned to basic event \( e \) as defined in (4).

\[
\mu_e^L(x) = \min\{\mu_{ij}(x)\}_{i=1; j=1}^{7n}
\]  

(4)

Step 1.3: Upper bound fuzzy probability calculation

The upper bound fuzzy probability of basic event \( e \) is generated using the highest fuzzy probability from the \( n \) fuzzy probabilities assigned to basic event \( e \) as defined in (5).

\[
\mu_e^U(x) = \max\{\mu_{ij}(x)\}_{i=1; j=1}^{7n}
\]  

(5)

If a fault tree of the safety system being evaluated has \( m \) basic events, then the \( m \) best estimate fuzzy probabilities, the \( m \) lower bound fuzzy probabilities, and the \( m \) upper bound fuzzy probabilities will then be generated in this step.

Step 2: Propagating fuzzy probabilities from basic events to the top event

The objective of this step is to quantify each subtree from leave nodes to the top event. A subtree is a simple fault tree which consists of two or more inputs \( (e_1, e_2, e_3, \ldots, e_n) \), one output \( (e_0) \) and one Boolean gate (an OR gate or an AND gate).

The output of an OR Boolean gate and an AND Boolean gate of a subtree can be quantified using a fuzzy complementation rule and a fuzzy multiplication rule, respectively, as shown in (6-7).

\[
\mu_{e_0}(x) = 1 - \prod_{i=1}^{n} \left\{ 1 - \mu_{e_i}(x) \right\}
\]  

(6)

\[
\mu_{e_0}(x) = \prod_{i=1}^{n} \mu_{e_i}(x)
\]  

(7)

where \( n \) is the number of input events.

In this step, equations (6-7) are repeatedly used to quantify sub trees from basic events throughout the system fault tree until the best estimate, lower bound and upper bound fuzzy probabilities of the top event, i.e., \( \mu_T^e(x) \), \( \mu_T^L(x) \), and \( \mu_T^U(x) \), are obtained.

Step 3: Converting the top event fuzzy probability into a probability

The objective of this step is to generate the best estimate, lower bound and upper bound probabilities to describe the failure of the top event, i.e. \( P_T^b \), \( P_T^l \) and \( P_T^u \) from its corresponding fuzzy probabilities \( \mu_T^e(x) \), \( \mu_T^L(x) \) and \( \mu_T^U(x) \), which have been generated in Step 2, using (8) [25].

\[
P_T = \begin{cases} 
1 - \frac{1}{10^1 \left( \frac{1-R_S}{R_S} \right)^{\frac{1}{3} \times 2.301}} & R_S \neq 0 \\
0 & R_S = 0 
\end{cases}
\]  

(8)

where \( R_S \) is the reliability score of the top event.

An area defuzzification technique has been specifically developed for NPP safety assessment by using fuzzy fault tree analysis to decode a fuzzy probability into a reliability score [26]. The \( R_S \) of fuzzy probability \( \mu(x) = (x_1, x_2, x_3) \) can be calculated using (9).

\[
R_S = \frac{1}{18} (4x_1 + x_2 + x_3)
\]  

(9)

where \( x_1, x_2 \) and \( x_3 \) are, respectively, the left support, the core, and the right support of the membership function representing a fuzzy probability.

Problem description

A large break LOCA is defined as a pipe rupture of a total cross-sectional area of equal to or greater than 0.09 \( m^2 \) of the reactor primary cooling system. In the AP1000, the three passive safety systems to mitigate the large break LOCA are an injection system by accumulator (AI), a low pressure injection system (LPI S) and a long term cooling system (LTC S) as shown by the event tree in Fig. 1 [9].

![Fig. 1. The event tree to mitigate the AP1000 large break LOCA.](image-url)
Figure 1 shows that a large break LOCA in the AP1000 can be successfully mitigated if the LTCS functions properly as expected. To successfully perform its function, the LTCS involves a residual heat removal system recirculation (RHR-SR) and a long term cooling passive system (LTCPS). Meanwhile, the success of the LTCPS depends on the success of the low pressure passive injection line (LPPI) and the containment cooling passive system (CCPS) [9]. The schematic diagram of the LTCS is shown in Fig. 2. Meanwhile, the schematic diagram of CCPS is given in Fig. 3. Due to the complexity of the LTCS fault tree, this fault tree is presented in Fig. 4 and Fig. 5. Since there are actually two lines in the LTCS but Fig. 2 only shows one line, $V_{kl}$ in Fig. 4 means the valve $V_k$ of the line $k$. Therefore, $V_{16}$ in Fig. 4 is the valve $V_6$ in Fig. 2 of the first line. Similarly, $V_{26}$ in Fig. 4 is the valve $V_6$ in Fig. 2 for the second line.

The epistemic uncertainty of the LTCS to mitigate the AP1000 large break LOCA is then evaluated using the proposed fuzzy probability algorithm described in the previous Section.

**RESULTS AND DISCUSSION**

This section mathematically illustrates the quantification process of the proposed fuzzy probability algorithm and discusses the results to confirm its feasibility and applicability to the evaluation of uncertainty in fault tree analysis.

**Step 1: Generating basic event fuzzy probabilities**

From the LTCS fault tree given in Figure 4, it can be seen that there are 26 basic events which need to be evaluated by experts by describing their failure possibilities using the set of seven failure possibilities given in Table 1. For simplification purposes, let us assume that seven experts with the
same level of expertise have been selected using the elicitation technique described in the previous section as denoted below.

\[ W = \left\{ w_i = \frac{1}{7} \mid i = 1, 2, \ldots, 7 \right\} \]

This number of experts has been properly implemented in nuclear power plant safety assessment involving experts [15,19]. Those seven selected experts are then asked to qualitatively characterize the occurrence likelihoods of those 26 basic events. For example, seven experts have characterized the occurrence likelihoods of basic event RHRS as Moderate, High, Reasonably High, High, High, Reasonably High. Therefore, the set of fuzzy probabilities describing the likelihood occurrences of basic event RHRS can be denoted as follows:

\[ \mu_{RHRS} = \{ \mu_M(x), \mu_H(x), \mu_{RH}(x), \mu_{HI}(x), \mu_H(x), \mu_{RH}(x) \} \]

The occurrence likelihoods of other basic events are given in Table 2 and their corresponding fuzzy probabilities are shown in Table 3.

**Table 2.** Failure possibilities of the LTCS fault tree evaluated by experts

<table>
<thead>
<tr>
<th>Basic events</th>
<th>Failure possibilities subjectively justified by expert</th>
</tr>
</thead>
<tbody>
<tr>
<td>V11</td>
<td>RL RL RL RL RL RL RL</td>
</tr>
<tr>
<td>V12</td>
<td>RL RL RL RL RL RL RL RL</td>
</tr>
<tr>
<td>V13</td>
<td>RL RL RL RL RL RL RL M</td>
</tr>
<tr>
<td>V14</td>
<td>RL RL RL RL RL RL RL M</td>
</tr>
<tr>
<td>V15</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V16</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V17</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V18</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V19</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V20</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V21</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V22</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V23</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V24</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V25</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V26</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V27</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V28</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>V29</td>
<td>RL RL RL RL RL RL M M M</td>
</tr>
<tr>
<td>RHRS</td>
<td>M H RH H H H RH</td>
</tr>
<tr>
<td>MOV</td>
<td>RL RL RL RL RL RL RL RL</td>
</tr>
<tr>
<td>MOV</td>
<td>RL RL RL RL RL RL RL RL</td>
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<td>MOV</td>
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<td>MOV</td>
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</tr>
<tr>
<td>MOV</td>
<td>RL RL RL RL RL RL RL RL</td>
</tr>
</tbody>
</table>

**Step 1.1: Best estimate fuzzy probability calculation**

Using (1), the best estimate fuzzy probability of basic event RHRS can be calculated as follows:

\[ \hat{\mu}_{RHRS} = \frac{1}{7} \times (\mu_M(x) + \mu_H(x) + \mu_{RH}(x) + \mu_H(x) + \mu_{RH}(x)) = (0.69, 0.78, 0.86). \]

Using the same procedure, the best estimate fuzzy probabilities for other basic events can be generated and the results are presented in Table 4.

**Step 1.2: Lower bound fuzzy probability calculation**

Using (4), the lower bound fuzzy probability of basic event RHRS is then generated as follows:

\[ \mu_{RHRS} = \min \left\{ \mu_{RHRS_i} \right\}_{i=1}^{7} = \mu_M(x). \]
Using the same procedure, the lower bound fuzzy probabilities for other basic events can be generated and the results are presented in Table 4.

### Table 4. Basic event lower bound, best estimate and upper bound fuzzy probabilities

<table>
<thead>
<tr>
<th>Basic events</th>
<th>Lower bound fuzzy probability</th>
<th>Best estimate fuzzy probability</th>
<th>Upper bound fuzzy probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td>$\mu_{v_1}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_1}^m(x) = (0.20, 0.30, 0.41)$</td>
<td>$\mu_{v_1}^U(x) = (0.33, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_2$</td>
<td>$\mu_{v_2}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_2}^m(x) = (0.20, 0.30, 0.41)$</td>
<td>$\mu_{v_2}^U(x) = (0.33, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_3$</td>
<td>$\mu_{v_3}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_3}^m(x) = (0.20, 0.30, 0.41)$</td>
<td>$\mu_{v_3}^U(x) = (0.33, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_4$</td>
<td>$\mu_{v_4}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_4}^m(x) = (0.25, 0.37, 0.49)$</td>
<td>$\mu_{v_4}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_5$</td>
<td>$\mu_{v_5}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_5}^m(x) = (0.25, 0.37, 0.49)$</td>
<td>$\mu_{v_5}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_6$</td>
<td>$\mu_{v_6}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_6}^m(x) = (0.22, 0.34, 0.45)$</td>
<td>$\mu_{v_6}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_7$</td>
<td>$\mu_{v_7}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_7}^m(x) = (0.30, 0.43, 0.57)$</td>
<td>$\mu_{v_7}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_8$</td>
<td>$\mu_{v_8}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_8}^m(x) = (0.20, 0.30, 0.41)$</td>
<td>$\mu_{v_8}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_9$</td>
<td>$\mu_{v_9}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_9}^m(x) = (0.20, 0.30, 0.41)$</td>
<td>$\mu_{v_9}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_{10}$</td>
<td>$\mu_{v_{10}}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_{10}}^m(x) = (0.25, 0.37, 0.49)$</td>
<td>$\mu_{v_{10}}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_{11}$</td>
<td>$\mu_{v_{11}}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_{11}}^m(x) = (0.25, 0.37, 0.49)$</td>
<td>$\mu_{v_{11}}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>$\mu_{v_{12}}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_{12}}^m(x) = (0.22, 0.34, 0.45)$</td>
<td>$\mu_{v_{12}}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_{13}$</td>
<td>$\mu_{v_{13}}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_{13}}^m(x) = (0.30, 0.43, 0.57)$</td>
<td>$\mu_{v_{13}}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_{14}$</td>
<td>$\mu_{v_{14}}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_{14}}^m(x) = (0.20, 0.30, 0.41)$</td>
<td>$\mu_{v_{14}}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
<tr>
<td>$v_{15}$</td>
<td>$\mu_{v_{15}}^L(x) = (0.17, 0.27, 0.37)$</td>
<td>$\mu_{v_{15}}^m(x) = (0.30, 0.43, 0.57)$</td>
<td>$\mu_{v_{15}}^U(x) = (0.35, 0.50, 0.65)$</td>
</tr>
</tbody>
</table>

### Step 1.3: Upper bound fuzzy probability calculation

Using (5), the upper bound fuzzy probability of basic event $RHS$, is generated as follows:

$$\mu_{RHS}^U(x) = \max \left\{ \mu_{RHS}^U(x) \right\}_{i=1}^7 = \mu_H(x).$$

Using the same procedure, the upper bound fuzzy probabilities for other basic events can be generated and the results are presented in Table 4.

The final results of this step are 26 best estimate fuzzy probabilities, 26 lower bound fuzzy probabilities and 26 upper bound fuzzy probabilities as shown in Table 4. Those values represent the occurrence likelihoods of the 26 basic event of the LTCS fault tree presented in Fig. 4.

### Step 2: Propagating fuzzy probabilities from basic events to the top event

This step is to illustrate how the output of the OR gate and the AND gate are to be quantified. For this purpose, the fuzzy probabilities of the intermediate event $Rec_{L_{11}} fail$ in Fig. 4 and the intermediate event $IL_1 fail$ in Fig. 5 are calculated. Each intermediate event will have a best estimate fuzzy probability, a lower bound fuzzy probability, and an upper bound fuzzy probability.

Using (6), the best estimate fuzzy probability of intermediate event $Rec_{L_{11}} fail$ is calculated as follows:

$$Rec_{L_{11}} fail = V_{16} + V_{17}$$

Using the same procedure and applying the lower bound and upper bound fuzzy probabilities of basic events $V_{16}$ and $V_{17}$, the lower bound and the upper bound fuzzy probabilities of intermediate event $Rec_{L_{11}} fail$ can be generated. The lower bound and the upper bound fuzzy probabilities of the $Rec_{L_{11}} fail$ are $\mu_{Rec_{L_{11}} fail}(x) = (0.3111, 0.4671, 0.6031)$ and $\mu_{Rec_{L_{11}} fail}^U(x) = (0.5775, 0.7500, 0.8775)$, respectively.

Meanwhile, using (7), the best estimate fuzzy probability of intermediate event $IL_1 fail$ is calculated as follows:

$$IL_1 fail = L_{11} fail \times L_{12} fail$$

Using the same procedure and applying the lower bound and upper bound fuzzy probabilities of intermediate events $L_{11} fail$ and $L_{12} fail$, the lower bound fuzzy probability and the upper bound fuzzy probability of intermediate event $IL_1 fail$ can be generated and those fuzzy probabilities are found to be $\mu_{IL_1 fail}^L(x) = (0.0968, 0.2182, 0.3637)$ and $\mu_{IL_1 fail}^U(x) = (0.3335, 0.5625, 0.7700)$, respectively.

By using (6) and (7) repeatedly to quantify the output of each Boolean gate in the LTCS fault tree, the best estimate, the lower bound, and the upper bound fuzzy probabilities of the other intermediate events, including the top event of the LTCS fault tree, can be generated as shown in Table 5.
Step 3: Converting the top event fuzzy probability into a probability.

From Table 5, it can be seen that the best estimate, the lower bound, and the upper bound fuzzy probabilities of the top event, which is the failure of the LTCS, are $\mu_{LTCS\ fail}^c(x) = (0.003168, 0.0032947, 0.141738)$, $\mu_{LTCS\ fail}^l(x) = (0.000322, 0.006967, 0.04758)$ and $\mu_{LTCS\ fail}^u(x) = (0.014871, 0.105506, 0.335265)$, respectively. Using (9), the $R_5$ of the top event fuzzy probability for the best estimate, lower bound and upper bound values are 0.010409, 0.003102 and 0.027792, respectively. Finally the best estimate, lower bound and upper bound probabilities to describe the failure of the top event can be generated using (8). Those three top event probabilities are $P_{LTCS\ fail}^c = 3.15 \times 10^{-11}$, $P_{LTCS\ fail}^l = 1.73 \times 10^{-10}$ and $P_{LTCS\ fail}^u = 2.98 \times 10^{-8}$. These values confirm that the best estimate probability to describe the failure of the LTCS to mitigate the AP1000 large break LOCAs is $3.15 \times 10^{-11}$ with the range of uncertainties is between $1.73 \times 10^{-10}$ and $2.98 \times 10^{-8}$. Meanwhile, Guimaraes et al. [9] calculated the best estimate failure probability of the LTCS to mitigate the AP1000 large break LOCAs is $1.11 \times 10^{-11}$ with the range of uncertainties between $2.21 \times 10^{-14}$ and $5.81 \times 10^{-9}$.

From those two results, it can be seen that the best estimate probability generated by the proposed fuzzy probability algorithm is very closed to the corresponding probability generated by Guimaraes et al. [9]. However, the uncertainty range generated by the proposed fuzzy probability algorithm is a bit wider than the one calculated by Guimaraes et al. [9]. This is because those two approaches apply different sources of basic event reliability data. Guimaraes et al. [9] assume that basic events have a point median value and an error factor, which are not always available. On the other hand, the proposed fuzzy probability algorithm assume basic events do not have probability distributions and rely on the expert judgments. However, to avoid wider uncertainty, experts involved to characterize basic event reliability have to be properly selected. In addition, uncertainty calculations...
by the proposed fuzzy probability algorithm is simpler than the one proposed by Guimaraes et al. [9]. The proposed fuzzy probability algorithm applies two simple fuzzy combination rules. Meanwhile, Guimaraes et al. [9] applies the $\alpha$-cut method, which is more complicated than fuzzy combination rules.

Nevertheless, the results of the case study have confirmed that the proposed fuzzy probability algorithm can propagate epistemic uncertainties from basic events to the top event and quantify fault trees.

**CONCLUSION**

The study presented in this article proposed a fuzzy probability algorithm to quantify epistemic uncertainty of the long term cooling system to mitigate the AP1000 large break LOCA when basic events do not have probability distributions. The best estimate failure probability of the AP1000 LTCS which is generated by the algorithm is $3.15 \times 10^{-11}$, which is very close to the reference value of $1.11 \times 10^{-11}$. Meanwhile, the uncertainty range of the fault tree generated by the proposed algorithm is between $1.16 \times 10^{-15}$ and $1.52 \times 10^{-8}$, which is a bit wider compared to the reference values, which are between $2.21 \times 10^{-14}$ and $5.81 \times 10^{-9}$. These results confirm that the proposed fuzzy probability algorithm is feasible for application in quantifying uncertainties in probabilistic safety assessment by fault tree analysis. Therefore, the proposed fuzzy probability algorithm is feasible for application in evaluating uncertainties in fault tree analysis when basic events do not have corresponding reliability data and experts are involved to evaluate the reliability characteristics of those basic events.

**REFERENCES**